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PREDICTIVE MODEL OF A PARACHUTE RETRACTION SOFT LANDING SYSTEM

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The U.S. Army Soldier Systems Command's Natick Research, Development and Engineering Center (NRDEC) is currently examining a novel concept for reducing the impact shock sustained by airdropped payloads upon ground impact. A device, called a parachute retractor, is placed between the payload and parachute confluence point, and when activated, accelerates the parachute and payload toward each other; slowing the payload prior to ground impact. The goal is to eliminate the cushioning material currently placed under airdrop loads, providing a roll-on/roll-off (RO/RO) capability. The retractor concept consists of a pneumatically driven piston/cylinder mechanism connected by cables to upper and lower pulley blocks to increase the system's overall mechanical advantage. Full scale testing of payload/retractor combinations is considered impractical, given the varied weights of military cargo presently airdropped and the multitude of retractor configurations possible. The need for a computational tool to determine the activation height and to optimize system design parameters, therefore, was recognized early on in the exploratory development effort. This report describes a predictive model, developed in response to that need, which couples a simplified parachute model to a model of the retractor mechanism. This model is able to predict the motion of the piston, payload and parachute confluence point, as well as the forces generated during retraction. This report first reviews the model's underlying theory and method of coupling. Computer program predictions are then compared to behavior observed in an experiment conducted on a instrumented prototype retractor device at Tustin Marine Corps Air Station, Santa Ana, CA in April 1994. The model, when validated, will provide the computational tools needed to support advanced development and eventual fielding of a Parachute Retraction Soft Landing System (PRSLS).

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Preface

The work described in this report was funded under the in-house work unit entitled, "Parachute Retraction Soft Landing Concept," PE 1L162786, PR D283, WU AA0C, AG CODE T/B 1415. The work was performed by the Applied Research Division (ARD) of the Mobility Directorate (MobD) during the period from April 1995 to September 1995. Since then, a reorganization of MobD has taken place. Inquiries concerning this effort should now be directed to MobD's Technical Management Office (TMO), Science & Technology Group (S&TG), Computational Analysis Team (CAT).

The author wishes to thank Mr. Steven E. Kunz of S&TG's Experimental Research Team (ERT), for his helpful suggestions concerning the thermodynamics of the retraction process, and providing the Tustin experimental data used in comparing model predictions with actual system behavior.

PREDICTIVE MODEL OF A PARACHUTE RETRACTION SOFT LANDING SYSTEM

Introduction

The U.S. Army Soldier Systems Command's Natick Research, Development and Engineering Center (NRDEC) is currently examining a concept for reducing the impact shock sustained by airdropped payloads upon ground impact. A device, called a parachute retractor, is placed between the payload and parachute confluence point, and when activated, accelerates the parachute and payload toward each other; slowing the payload prior to ground impact. The goal is to eliminate the cushioning material currently placed under airdrop loads, and thus eliminate the time-consuming, labor-intensive process of rigging payloads for airdrop, while providing a roll-on/roll-off (RO/RO) capability for vehicles.

The concept of parachute retraction to soft land cargo is not new. Its origins date back to a 1956 report on airdrop cushioning prepared by the University of Texas for NRDEC, then known as the Quartermaster Research and Development Command[1]. In that report, the characteristics of an elastic spring-type device for use with parachutes, dubbed a landing snubber, were described and a theoretical analysis of its operation presented. A further expansion of the concept, known as Parachute Reel-In Reel-Out, was examined by NRDEC in the 1960's. In addition to soft landing cargo, this concept envisioned use of retraction/unreeling of the lines connecting the parachutes and payload in order to maintain high relative velocities of the airstream with respect to the parachutes, and to relieve unwanted high forces on the cargo at appropriate times during airdrop descent. Use of a powered winching device located between the recovery parachutes and the cargo to either decrease or increase the distance between the parachutes and cargo during the trajectory was envisioned. None of these concepts ever reached the prototype stage or were tested.

In the spring of 1994, instrumented airdrop tests of a novel retraction device invented by John Lanza of Natick were conducted for the first time at Tustin Marine Corps Air Station (MCAS), Santa Ana, CA. Experimental data obtained in these tests showed that the Natick parachute retractor could slow or even stop payloads just before landing. The retractor consisted of a pneumatically driven piston connected by cables to upper and lower pulley blocks; all enclosed within a cylinder. The pulleys are used to increase the length of retraction compared with

the stroke of the piston, as is shown schematically in Figure 1.

The benefit of developing a model, which can predict the performance of this Parachute Retraction Soft Landing System (PRSLS) for all airdrop configurations, including large, yet to be investigated (experimental) systems, was recognized early in the program. Therefore, as part of this exploratory development effort, a computational model was developed to determine the optimal activation height, to conduct system trade-off studies, and to investigate scaling effects. This report reviews the development of that computer model.

The report begins with a review of the model's underlying theory and assumptions. Applying the principles of thermodynamics, kinetics, kinematics, fluid mechanics and numerical methods, key mathematical relationships used in modeling the system as a three-mass, three-degree-of-freedom system are first derived. Derivations are given exhaustive treatment in order to enhance the reader's understanding of the theory, principles, and assumptions that provide the foundation for model development. Rigorous treatment is given to the development of expressions for the forcing funtions, i.e., force acting on the piston, the work done by the expanding gas on the piston, the differential equations of motion of the discrete masses, the governing nonlinear differential equation of motion of the system, and the recurrence formulas used to integrate the motion equation. Solution of the equation of motion and the trial and error iterative scheme used to reconcile the system's thermodynamics and kinetics and arrive at the time values are discussed next. Finally, FORTRAN computer model predictions are compared to the data obtained during experiments conducted at Tustin MCAS on 20 April 1994, and conclusions are drawn concerning model accuracy.

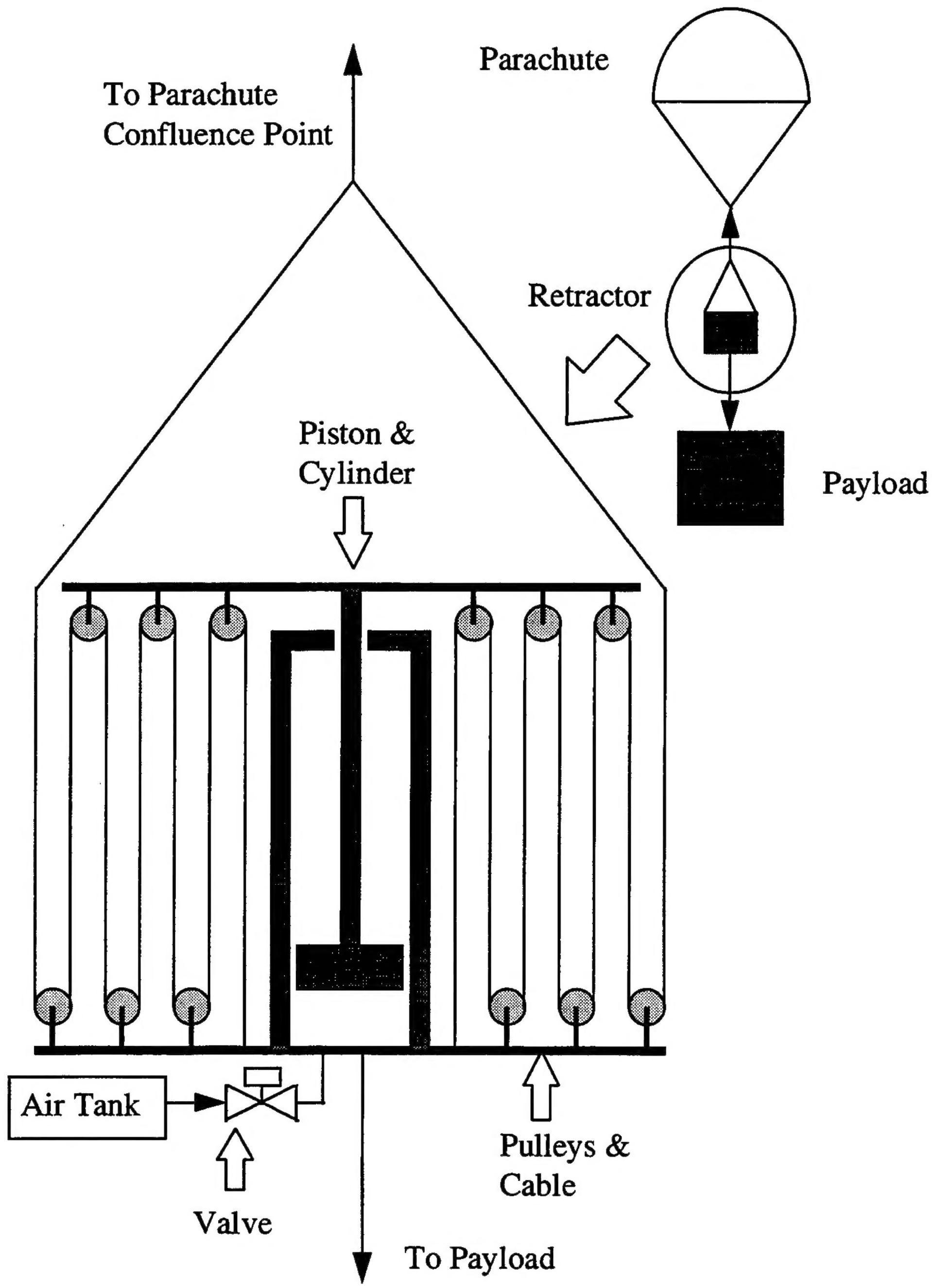

Figure 1: Schematic Diagram of Parachute Retractor

Thermodynamics

The principles of thermodynamics[2,3] are applied in this section to the development of expressions for the force exerted on the piston, and the work produced by the expanding gas, upon valve opening. The piston/cylinder mechanism is assumed to be frictionless. Force and work expressions are derived as a function of piston displacement, y, for the processes of both adiabatic and isothermal ideal gas expansion within the cylinder. Constants include the tank volume, V_i , initial tank pressure, P_i , specific heat ratio, k, (adiabatic only) and piston radius, r. Derivations for each process begin with an examination of the First Law of Thermodynamics, which in differential form, is written as follows:

$$dQ=dW+dU$$

This states that the heat entering the system, dQ, is equivalent to the work done against the atmosphere, dW, plus the internal energy accumulation, dU.

Adiabatic Expansion Process

An ideal gas expands from initial state (1) to final state (2) in a cylinder which is completely insulated so that there is no heat exchange with the surroundings; that is: Q=0.

The First Law of Thermodynamics in differential form is written as:

$$dQ=dW+dU=0$$

or

$$dW_q=-dU$$

which indicates that the work effect is produced at the expense of the system's internal energy.

For an ideal gas, the enthalpy, H, which is equal to U+PV, and internal energy, U, depend on the temperature only and are independent of pressure.

Therefore, the following relationships hold true at all pressures:

$$\Delta H_{Ideal\,Gas}=\int_{T_1}^{T_2} c_p dT$$

$$\Delta U_{Ideal\,Gas}=\int_{T_1}^{T_2} c_v dT$$

where c_p and c_v are respectively the specific heats at constant pressure and constant volume, and R, the universal gas constant, is equal to c_p-c_v. It should be apparent from the above relationships that for isothermal processes (i.e., constant temperature, ΔT=0), both the change in enthalpy, ΔH, and internal energy change, ΔU, are equal to zero. Returning to the adiabatic case, it follows that for gas expansion within the cylinder:

$$W_q=-U=-nc_v(T_2-T_1)$$

or

$$dW_q=PdV=-nc_v dT$$

where n is the number of moles of gas present. Upon differentiating the Ideal Gas Law equation:

$$PV=nRT$$

one obtains the following:

$$PdV+VdP=nRdT$$

Upon substituting ($-PdV/nc_v$) for dT in the above equation one obtains:

$$PdV+VdP=-nR\left(\frac{PdV}{nc_v}\right)$$

Rearranging terms results in the following expression:

$$\left(1+\frac{R}{c_v}\right)PdV=-VdP$$

But $R=c_p-c_v$. Therefore:

$$\left(\frac{c_p}{c_v}\right)\frac{dV}{V}=-\frac{dP}{P}$$

Integrating between states (1) and (2):

$$\int_{V_1}^{V_2}\left(\frac{c_p}{c_v}\right)\frac{1}{V}dV=\int_{P_1}^{P_2}-\frac{1}{P}dP$$

$$\left(\frac{c_p}{c_v}\right)\ln V\,\Big|_{V_1}^{V_2}=-\ln P\,\Big|_{P_1}^{P_2}$$

$$\left(\frac{c_p}{c_v}\right)\left(\ln V_2-\ln V_1\right)=\ln P_1-\ln P_2$$

$$\left(\frac{c_p}{c_v}\right)\ln\left(\frac{V_2}{V_1}\right)=\ln\left(\frac{P_1}{P_2}\right)$$

$$\ln\left(\frac{V_2}{V_1}\right)^{\left(\frac{c_p}{c_v}\right)}=\ln\left(\frac{P_1}{P_2}\right)$$

$$\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^k$$

where $k = c_p / c_v$.

$$\therefore\ P_1V_1^k = P_2V_2^k$$

It should be noted that for a Monatomic Gas:

c_p = 5/2 R
c_v = 3/2 R

and for a Diatomic Gas:

c_p = 7/2 R
c_v = 5/2 R

Since $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$, upon substitution of nRT_1 / V_1 and nRT_2 / V_2 respectively for P_1 and P_2 in the above equation one obtains:

$$\left(\frac{nRT_1}{V_1}\right)V_1^{\,k} = \left(\frac{nRT_2}{V_2}\right)V_2^{\,k}$$

$$T_1V_1^{\,k-1} = T_2V_2^{\,k-1}$$

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \qquad (1)$$

Similarly, if one substitutes nRT_1 / P_1 and nRT_2 / P_2 respectively for V_1 and V_2 one obtains:

$$P_1\left(\frac{nRT_1}{P_1}\right)^k = P_2\left(\frac{nRT_2}{P_2}\right)^k$$

$$\frac{T_1^kP_1}{P_1^k} = \frac{T_2^kP_2}{P_2^k}$$

$$\frac{T_1^k}{P_1^{k-1}}=\frac{T_2^k}{P_2^{k-1}}$$

$$\left(\frac{T_2}{T_1}\right)^k=\left(\frac{P_2}{P_1}\right)^{k-1}$$

$$\therefore\quad \frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad (2)$$

The adiabatic Volume-Temperature and Pressure-Temperature relationships shown respectively in (1) and (2) were not incorporated into the model. These equations were derived and are presented nonetheless, since they would be needed to develop expressions to predict the gas temperature from valve opening up to the full piston stroke, if desired at some later time.

Given an initial tank pressure, P_i , and tank volume, V_i , the gas volume at some later time during the expansion is given by the expression:

$$V(y)=V_i+\pi r^2 y$$

where r is the Piston Radius and y is the Piston Displacement.

From the previously derived Pressure-Volume relationship for adiabatic ideal gas expansion it is evident that:

$$P_i V_i^k=P(y)V(y)^k$$

Upon substitution of the above expression for V(y) one obtains the following:

$$P(y)=\frac{P_i V_i^k}{\left(V_i+\pi r^2 y\right)^k}$$

Noting that the force, F, acting on the piston is equal to $\pi r^2 P(y)$ it follows that:

$$F(y)=\frac{\pi r^2 P_i V_i^k}{\left(V_i+\pi r^2 y\right)^k} \qquad (3)$$

But $dV/dy = \pi r^2$ or $dV = \pi r^2 dy$. Therefore, the work done on the piston during the adiabatic expansion of the gas within the cylinder is given by the following:

$$W_q=\int_0^S PdV=\int_0^S \frac{P_i V_i^k}{\left(V_i+\pi r^2 y\right)^k}\pi r^2 dy$$

where S is the piston stroke and $0 \leq y \leq S$.

Upon collecting and rearranging terms, one obtains the following expression:

$$W_q=\pi r^2 P_i V_i^k \int_0^S \frac{1}{\left(V_i+\pi r^2 y\right)^k}dy$$

or

$$W_q=\pi r^2 P_i V_i^k \int_0^S \left(V_i+\pi r^2 y\right)^{-k}dy$$

Let $u = V_i + \pi r^2 y$. Then $du = \pi r^2 dy$ or $dy = du / \pi r^2$. Upon substitution into the above equation for W_q one obtains the following:

$$W_q=\pi r^2 P_i V_i^k \int_0^S u^{-k}\frac{du}{\pi r^2}=P_i V_i^k \int_0^S u^{-k}du$$

Integrating the above expression results in:

$$W_q=P_i V_i^k \left(\frac{u^{1-k}}{1-k}\right)\Bigg|_0^S$$

Substituting $V_i + \pi r^2 y$ for u in the above equation results in:

$$W_q = P_i V_i^k \frac{\left(V_i + \pi r^2 y\right)^{1-k}}{(1-k)}\Bigg|_0^S$$

Evaluating the above integral results in the following equation:

$$W_q = P_i V_i^k \frac{\left(V_i + \pi r^2 S\right)^{1-k}}{1-k} - \left(\frac{P_i V_i}{1-k}\right)$$

$$\therefore\ W_q = \frac{P_i V_i^k \left(V_i + \pi r^2 S\right)^{1-k} - P_i V_i}{(1-k)}$$

The total adiabatic work done by the expanding gas up to some displacement, y, during piston movement, is similarly obtained by evaluating the above integral between zero and the value of y. Thus:

$$W_q(y) = \frac{P_i V_i^k \left(V_i + \pi r^2 y\right)^{1-k} - P_i V_i}{(1-k)} \qquad \textbf{(4)}$$

where $0 \leq y \leq S$.

Isothermal Expansion Process

An ideal gas expands from initial state (1) to final state (2) in a cylinder. The temperature remains constant throughout the process. The volume of the gas increases as the pressure of the gas decreases. The cylinder is in contact with a constant temperature bath which adds heat to the system as the system does expansion work.

The First Law of Thermodynamics requires that:

$$dQ_T = dW + dU$$

Since temperature is constant, i.e., dT=0, dU of the ideal gas is equal to zero, and all the heat entering the system leaves as work. Therefore:

$$dQ_T = dW$$

and the Ideal Gas Law in differential form is:

$$PdV + VdP = 0$$

Therefore:

$$\frac{dV}{V} = -\frac{dP}{P}$$

Integrating between states (1) and (2);

$$\int_{V_1}^{V_2} \frac{1}{V} dV = \int_{P_1}^{P_2} -\frac{1}{P} dP$$

$$\ln V \,\Big|_{V_1}^{V_2} = -\ln P \,\Big|_{P_1}^{P_2}$$

$$(\ln V_2 - \ln V_1) = \ln P_1 - \ln P_2$$

$$\ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{P_1}{P_2}\right)$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\therefore\ P_1 V_1 = P_2 V_2$$

Given an initial tank pressure, P_i , and tank volume, V_i , the gas volume at some later time during the expansion is given by the expression:

$$V(y) = V_i + \pi r^2 y$$

where r is the Piston Radius and y is the Piston Displacement.

From the Ideal Gas Law for a constant temperature process, it is evident that:

$$P_i V_i = P(y)V(y)$$

Substitution of the previously derived expression for V(y) into the isothermal Ideal Gas relationship results in the following:

$$P(y) = \frac{P_i V_i}{\left(V_i + \pi r^2 y\right)}$$

Noting that the force, F, acting on the piston is equal to $\pi r^2 P(y)$ it follows that:

$$F(y) = \frac{\pi r^2 P_i V_i}{\left(V_i + \pi r^2 y\right)} \qquad (5)$$

From the volume expression it follows that $dV/dy = \pi r^2$ or $dV = \pi r^2 dy$. Therefore, the work done on the piston during the isothermal expansion of the gas within the cylinder is given by the following:

$$W_T = \int_0^S P dV = \int_0^S \frac{P_i V_i}{\left(V_i + \pi r^2 y\right)} \pi r^2 dy$$

$$\therefore \; W_T = P_i V_i \pi r^2 \int_0^S \left(V_i + \pi r^2 y\right)^{-1} dy$$

where S is the Piston Stroke and $0 \leq y \leq S$.

Let $u = V_i + \pi r^2 y$. Then $du = \pi r^2 dy$ or $dy = du / \pi r^2$. Upon substitution into the above equation for W_T one obtains the following:

$$W_T = P_i V_i \pi r^2 \int_0^S \frac{1}{u} \frac{du}{\pi r^2} = P_i V_i \ln u \;\Big|_0^S$$

Substituting the expression for u back into the above equation and evaluating the integral results in the following:

$$W_T=P_iV_i\ln(V_i+\pi r^2y)\Big|_0^S=P_iV_i\left[\ln(V_i+\pi r^2S)-\ln V_i\right]$$

$$\therefore\ W_T=P_iV_i\ln\left(\frac{V_i+\pi r^2S}{V_i}\right)$$

The total isothermal work done by the expanding gas up to some displacement, y, during piston movement, is similarly obtained by evaluating the above integral between zero and the value of y. Thus:

$$W_T(y)=P_iV_i\ln\left(\frac{V_i+\pi r^2y}{V_i}\right) \qquad (6)$$

The above work expression can be derived alternatively by noting that the Ideal Gas expression PV = nRT permits the elimination of either P or dV from the work expression:

$$W=\int_1^2 PdV$$

From the Ideal Gas Equation it is apparent that:

$$P=\frac{nRT}{V}=\frac{RT}{\bar{V}}$$

or

$$d\bar{V}=\frac{-RT}{P^2}dP+\frac{R}{P}dT$$

where

$$\bar{V}=\frac{V}{n}$$

is defined as the volume per mole or molar volume.

Upon substitution of the above expressions into the work equation, one obtains the following:

$$W=\int RT\frac{d\bar{V}}{\bar{V}}$$

or

$$W=\int\left(\frac{-RT}{P}dP+RdT\right)$$

But dT = 0. Therefore:

$$W_T=nRT\int_1^2\frac{dV}{V}=nRT\ln\left(\frac{V_2}{V_1}\right)$$

or

$$W_T=-nRT\int_1^2\frac{dP}{P}=nRT\ln\left(\frac{P_1}{P_2}\right)$$

Let $P_1V_1 = nRT = P_iV_i$. One will note that the judicious substitution of P_i, V_i and the previously derived expressions for V(y) and P(y) respectively for P_1, V_1, V_2 and P_2 in either of the above work expressions results in the previously derived work equation (4) for isothermal gas expansion in a cylinder.

Development of the Differential Equations of Motion

The PRSLS was modeled as a three mass, three degree-of-freedom (DOF) rigid body system. Assumptions made in order to simplify modeling of the system included the following:

(1) Piston and pulley movement is frictionless.
(2) Pulley cables are inextensible, i.e., cable length remains constant.
(3) System orientation remains vertical throughout the retraction.
(4) The apparent mass of the parachute, which consists of air inside the canopy, as well as that in the immediate vicinity, is constant and equal to the mass of air contained within a hemisphere of nominal parachute diameter.
(5) The parachute drag coefficient, based on the nominal parachute diameter, remains constant during retraction.

The PRSLS model, as previously noted, is comprised of three discrete masses. They include: (1) the piston and any upper retraction device cross members and upper pulley blocks that move along with the piston; defined as M_{PIST} , (2) the payload and the retraction device cylinder, tank, lower pulley blocks and structure that move with the payload; defined as M_P , and (3) the parachute with mass concentrated at the parachute confluence point; defined as M_C. The three degrees-of-freedom (DOF) considered are the vertical, i.e., Y-direction, displacements of the piston, payload and parachute confluence point; defined respectively as y, y_P and y_C.

The retraction device modeled contains two banks of upper and lower pulleys, and two sets of pulley cables. Each of the pulley cables, which converge at the parachute confluence point, by symmetry carry one-half the tension, T_C, at the parachute confluence point at any instant in time during the system's descent, just prior to and during retraction. Formulation of the differential equations of motion was carried out by applying the concept of dynamic equilibrium or Newton's second law of motion to the system[4]. For this purpose, the three masses were first isolated showing all of the forces acting on each mass, including internal forces, externally applied forces and inertia forces. A detailed examination of the forces acting on the piston and payload masses, i.e., M_{PIST} and M_P , revealed two important factors concerning internal forces developed within the retractor's pulley subsystem. First, for a pulley arrangement of a given mechanical advantage, N, the force on the piston tending to resist piston movement is equal to $2 \times N \times (T_C/2)$

$= N \times T_C$. Second, for a mechanical advantage, N, the force tending to accelerate the payload upward is equal to $2 \times (N+1) \times (T_C/2) = (N+1) \times T_C$.

Applying Newton's second law of motion, the equation of equilibrium for the piston in the vertical direction, $+\uparrow \Sigma F_v = ma$, is written as:

$$F - NT_C - W_{PIST} = M_{PIST}\ddot{y} \qquad (7a)$$

Similarly, the equilibrium equation for the payload in the vertical direction, $+\uparrow \Sigma F_v = ma$, is written as:

$$(N+1)T_C - F - W_P = M_P\ddot{y}_P \qquad (7b)$$

The force, F, in the above equations of motion, varies with time, i.e., F = F(t), and is the force applied to the piston by the expanding gas upon valve opening.

Finally, applying the principles of dynamic equilibrium at the parachute confluence point in the vertical direction, $+\downarrow \Sigma F_v = ma$, results in the following equilibrium equation:

$$T_C + W_C - F_{Parachute} = M_C\ddot{y}_C \qquad (7c)$$

The parachute drag force and apparent mass terms, $F_{Parachute}$ and M_C , in the above equation are given by the expressions[5]:

$$F_{Parachute} = \frac{\pi r_P^2 C_D \rho}{2}(\dot{y}_C)^2 \qquad (8)$$

and

$$M_C = \frac{2}{3}\pi \rho r_P^3 \qquad (9)$$

where r_P is the nominal parachute radius, which is equal to one-half the nominal parachute diameter, C_D is the drag coefficient based on the total canopy surface area, and ρ is the density of air. In order to simplify analysis, the parachute model therefore assumes that both the apparent mass and drag coefficient remain constant during retraction.

If one defines the constant, C_1 , where:

$$C_1 = \frac{\pi r_P^2 C_D \rho}{2} \qquad (10)$$

Then it follows that:

$$F_{Parachute} = C_1 (\dot{y}_C)^2 \qquad (11)$$

Substituting the above expression for $F_{Parachute}$ into (7c) and noting the weight of the air in the parachute, W_C , is negligible and can be ignored, one obtains the following:

$$T_C = C_1 (\dot{y}_C)^2 + M_C \ddot{y}_C \qquad (12)$$

Substituting the above expression for T_C into (7a) and (7b) results in the following expressions:

$$F - NC_1 (\dot{y}_C)^2 - NM_C \ddot{y}_C - W_{PIST} = M_{PIST} \ddot{y} \qquad (13a)$$

$$(N+1) C_1 (\dot{y}_C)^2 + (N+1) M_C \ddot{y}_C - F - W_P = M_P \ddot{y}_P \qquad (13b)$$

Dividing both sides of (13a) and (13b) respectively by M_{PIST} and M_P result in the following expressions for the piston and payload accelerations:

$$\ddot{y} = \frac{\left[F - NC_1 (\dot{y}_C)^2 - NM_C \ddot{y}_C - W_{PIST}\right]}{M_{PIST}} \qquad (14a)$$

$$\ddot{y}_P = \frac{\left[(N+1) C_1 (\dot{y}_C)^2 + (N+1) M_C \ddot{y}_C - F - W_P\right]}{M_P} \qquad (14b)$$

The next step is to define the retraction, retraction rate, and retraction acceleration in terms of the displacements, velocities, and accelerations at the piston, payload and parachute confluence point. Based on the sign convention used in deriving the differential equations of motion, the following relationships are defined:

$$L = Ny = y_C + y_P \qquad (15a)$$

$$\dot{L} = N\dot{y} = \dot{y}_C + \dot{y}_P \qquad (15b)$$

$$\ddot{L} = N\ddot{y} = \ddot{y}_C + \ddot{y}_P \qquad (15c)$$

Therefore, it is evident that:

$$\ddot{y}_P = N\ddot{y} - \ddot{y}_C$$

Substituting the above expression for the payload acceleration into (13b) and dividing both sides of (13a) and (13b) respectively by M_{PIST} and NM_P results in the following expressions upon collecting and rearranging terms:

$$\frac{F}{M_{PIST}} - \frac{NC_1}{M_{PIST}}(\dot{y}_C)^2 - \frac{NM_C}{M_{PIST}}\ddot{y}_C - \frac{W_{PIST}}{M_{PIST}} = \ddot{y} \qquad (16a)$$

$$\frac{(N+1)C_1}{NM_P}(\dot{y}_C)^2 + \left[\frac{(N+1)M_C + M_P}{NM_P}\right]\ddot{y}_C - \frac{F}{NM_P} - \frac{W_P}{NM_P} = \ddot{y} \qquad (16b)$$

Equating (16a) and (16b) results in the following expression:

$$\frac{F}{M_{PIST}} - \frac{NC_1}{M_{PIST}}(\dot{y}_C)^2 - \frac{NM_C}{M_{PIST}}\ddot{y}_C - g = \frac{(N+1)C_1}{NM_P}(\dot{y}_C)^2 + \left[\frac{(N+1)M_C + M_P}{NM_P}\right]\ddot{y}_C - \frac{F}{NM_P} - \frac{g}{N}$$

where $M_{PIST}g$ and M_Pg have been substituted respectively for W_{PIST} and W_P, cancelling out the mass term in the last entry on both sides of the expression.

The following nonlinear differential equation of motion for the parachute retraction soft landing system is obtained upon collecting and rearranging terms in the previous expression:

$$\left[\frac{(N+1)C_1}{NM_P}+\frac{NC_1}{M_{PIST}}\right](\dot{y}_C)^2+\left[\frac{(N+1)M_C+M_P}{NM_P}+\frac{NM_C}{M_{PIST}}\right]\ddot{y}_C+\left(g-\frac{g}{N}\right)=F\left(\frac{1}{M_{PIST}}+\frac{1}{NM_P}\right)$$

Multiplying both sides of the equation shown above by NM_PM_{PIST} results in the following motion equation:

$$\left[(N+1)C_1M_{PIST}+N^2C_1M_P\right](\dot{y}_C)^2+\left[(N+1)M_CM_{PIST}+M_PM_{PIST}+N^2M_CM_P\right]\ddot{y}_C$$

$$+\left[NM_PM_{PIST}g-M_PM_{PIST}g\right]=F\left[NM_P+M_{PIST}\right] \qquad (17)$$

Define the following constants:

$$C_2=(N+1)C_1M_{PIST}+N^2C_1M_P$$

$$C_3=(N+1)M_CM_{PIST}+M_PM_{PIST}+N^2M_CM_P \qquad (18)$$

$$C_4=NM_PM_{PIST}g-M_PM_{PIST}g=(N-1)M_PM_{PIST}g$$

$$C_5=NM_P+M_{PIST}$$

Then the nonlinear differential equation of motion shown in (17) can be expressed as follows:

$$C_2(\dot{y}_C)^2+C_3\ddot{y}_C+C_4=C_5F$$

or

$$\ddot{y}_C=\frac{\left[C_5F-C_2(\dot{y}_C)^2-C_4\right]}{C_3} \qquad (19)$$

Linear-Acceleration Method

Integration of the nonlinear differential equation of motion developed in the previous section was carried out using the Linear-Acceleration method[6,7]. As implied by its name, this method assumes that the acceleration varies linearly between time stations as shown in Figure 2.

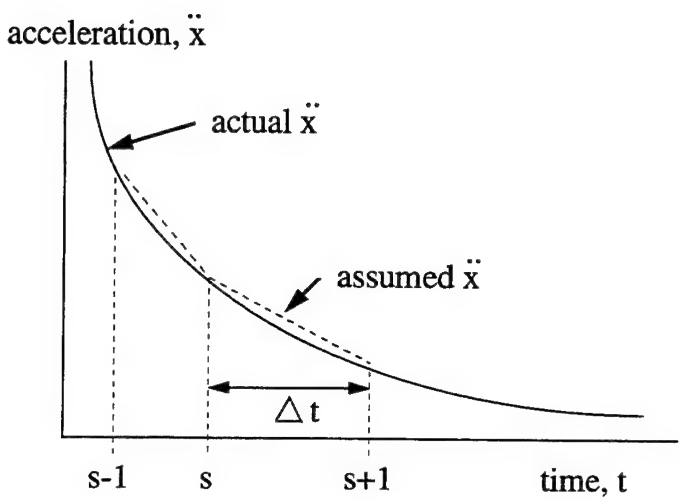

Figure 2: Linear-Acceleration Method; Actual vs. Assumed Acceleration

The acceleration between time stations s and s+1 would therefore be approximated by the expression:

$$\ddot{x}=\ddot{x}^{(s)}+\frac{\ddot{x}^{(s+1)}-\ddot{x}^{(s)}}{\Delta t}\left(t-t^{(s)}\right)$$

where the superscript enclosed by parentheses, by definition, represents the particular time station.

The velocity at any time within this interval may be obtained from the following:

$$\dot{x}=\dot{x}^{(s)}+\int_{t^{(s)}}^{t}\ddot{x}dt$$

or

$$\dot{x}=\dot{x}^{(s)}+\ddot{x}^{(s)}\left(t-t^{(s)}\right)+\frac{\ddot{x}^{(s+1)}-\ddot{x}^{(s)}}{2\Delta t}\left(t-t^{(s)}\right)^{2}$$

which at station s+1 becomes:

$$\dot{x}^{(s+1)}=\dot{x}^{(s)}+\frac{\Delta t}{2}\left(\ddot{x}^{(s+1)}+\ddot{x}^{(s)}\right)$$

The displacement at s+1 is given by:

$$x^{(s+1)}=x^{(s)}+\int_{t^{(s)}}^{t^{(s+1)}}\dot{x}dt$$

or

$$x^{(s+1)}=x^{(s)}+\dot{x}^{(s)}\Delta t+\frac{(\Delta t)^{2}}{6}\left(2\ddot{x}^{(s)}+\ddot{x}^{(s+1)}\right)$$

where from the general velocity expression:

$$\dot{x}^{(s)}=\dot{x}^{(s-1)}+\frac{\Delta t}{2}\left(\ddot{x}^{(s)}+\ddot{x}^{(s-1)}\right) \qquad (20)$$

The recurrence formula shown above can be expressed as follows:

$$\dot{x}^{(s)}=A^{(s-1)}+\ddot{x}^{(s)}\frac{\Delta t}{2} \qquad (21)$$

where

$$A^{(s-1)}=\dot{x}^{(s-1)}+\ddot{x}^{(s-1)}\frac{\Delta t}{2} \qquad (22)$$

From the previous discussion it can similarly be reasoned that:

$$x^{(s)}=x^{(s-1)}+\dot{x}^{(s-1)}\Delta t+\frac{(\Delta t)^2}{6}\left(2\ddot{x}^{(s-1)}+\ddot{x}^{(s)}\right) \qquad (23)$$

This equation may be expressed in terms of the acceleration at time station s as follows:

$$\ddot{x}^{(s)}=\frac{6\left(x^{(s)}-x^{(s-1)}\right)}{(\Delta t)^2}-\frac{6\dot{x}^{(s-1)}}{\Delta t}-2\ddot{x}^{(s-1)}$$

Substituting the above expression for the acceleration at time station s into the previous equation for the velocity at time station s results in the following:

$$\dot{x}^{(s)}=\dot{x}^{(s-1)}+\frac{3}{\Delta t}\left(x^{(s)}-x^{(s-1)}\right)-3\dot{x}^{(s-1)}-\Delta t\ddot{x}^{(s-1)}+\frac{\Delta t}{2}\ddot{x}^{(s-1)}$$

or

$$\dot{x}^{(s)}=\frac{3x^{(s)}}{\Delta t}-\frac{3x^{(s-1)}}{\Delta t}-2\dot{x}^{(s-1)}-\frac{\Delta t}{2}\ddot{x}^{(s-1)}$$

and

$$\frac{3x^{(s)}}{\Delta t}=\frac{3x^{(s-1)}}{\Delta t}+2\dot{x}^{(s-1)}+\frac{\Delta t}{2}\ddot{x}^{(s-1)}+\dot{x}^{(s)}$$

Therefore

$$x^{(s)}=x^{(s-1)}+\left(\frac{\Delta t}{3}\right)2\dot{x}^{(s-1)}+\frac{(\Delta t)^2}{6}\ddot{x}^{(s-1)}+\left(\frac{\Delta t}{3}\right)\dot{x}^{(s)}$$

The above recurrence formula can be expressed as:

$$x^{(s)}=B^{(s-1)}+\dot{x}^{(s)}\frac{\Delta t}{3} \qquad (24)$$

where

$$B^{(s-1)}=x^{(s-1)}+2\dot{x}^{(s-1)}\frac{\Delta t}{3}+\ddot{x}^{(s-1)}\frac{(\Delta t)^2}{6} \qquad (25)$$

In applying this method, equations (21) and (24) are used in succession. Since the value of the acceleration at time station s is not known in advance, the solution must be iterative within each step. This iterative process is not self-starting, however, since it requires a supplementary formula to determine the first estimate of the velocity at time station s in each time step. First estimates of this velocity were obtained using the formula:

$$\dot{x}^{(s)}=\dot{x}^{(s-1)}+\ddot{x}^{(s-1)}\Delta t \qquad (26)$$

in which the acceleration is assumed constant during the time interval and equal to the initial value.

Integration and Solution of the Nonlinear Differential Equation of Motion

Using the thermodynamic and kinetic relationships and numerical recurrence formulas derived in the previous sections, a FORTRAN computer model was developed to perform a time history analysis of the Parachute Retraction Soft Landing System (PRSLS). This model uses a two-step iterative procedure to integrate the nonlinear differential equation of motion in time; providing a time history of the motion of the piston, payload and parachute confluence point, i.e., displacements, velocities and accelerations, force exerted on the piston by the expanding gas, gas expansion work, line tension, parachute force, and retraction, retraction rate and retraction acceleration from valve opening up through full piston stroke. The FORTRAN coding for this computer model is listed in Appendix A of this report.

Model inputs include the initial tank pressure, tank volume, piston radius, specific heat ratio, weight of piston including cross member and top pulleys, payload weight including retractor, piston stroke, step increment of piston displacement at which time history values are desired, gas expansion process, i.e., adiabatic or isothermal, parachute diameter, mechanical advantage of pulley system, and parachute drag coefficient.

Using the above model inputs, the program begins the analysis by first computing the three mass terms, the terminal descent velocity of the system, and the constant terms used in the differential equation of motion. The mass at the parachute confluence point is computed using equation (9). Values of the constant terms, C1 through C5, are computed using equation (10) and equations (18). The terminal descent velocity of the PRSLS is computed from the well known parachute expression:

$$v_{TERM}=\sqrt{\frac{2\left(W_P+W_{PIST}\right)}{\pi r_P^2 C_D \rho}} \qquad (27)$$

where W_P, W_{PIST}, r_P, C_D, and ρ are as previously defined in this report.

The program next computes the initial conditions at time equals zero, i.e., when the valve opens and retraction begins. Gas expansion work, piston velocity

and the displacements of the piston, payload, and parachute confluence point are all set equal to zero at time equals zero. Although the payload and parachute confluence point are both initially assumed to be moving downward at terminal descent velocity, as an acknowledgment of the sign convention used in model development, their respective velocities are taken to be equal to plus and minus the value computed using equation (27). The force exerted on the piston by the expanding gas is computed next using either equation (3) or equation (5), depending upon whether adiabatic or isothermal gas expansion is assumed in the analysis. Using the model inputs supplied, and the values computed for the constant terms, the force acting on the piston, and the displacements and velocities of the three masses, the program is now able to compute all of the remaining forces, displacements, velocities and accelerations at time zero using previously derived expressions. The accelerations of the parachute confluence point, piston and payload are first computed using equations (19), (14a), and (14b) respectively. With the motion of the three masses now completely described at time equals zero, the remaining values for the line tension, parachute force, retraction, retraction rate, and retraction acceleration can then be computed respectively using equations (12), (11), (15a), (15b), and (15c).

Integration of the PRSLS's nonlinear differential equation of motion in time begins by first computing the piston force and gas expansion work when the piston has moved a distance, Δy, the step increment chosen by the program user. Force and work values are computed using either equations (3) and (4) or equations (5) and (6), depending upon whether adiabatic or isothermal gas expansion has been assumed in the analysis. Using recurrence formulas analogous to those shown in equations (20) through (26) and the differential equation of motion, equation (19), the displacement, velocity, and acceleration of the parachute confluence point are determined iteratively using the following procedure.

Assuming a value for the change in time, Δt, equal to 0.1 seconds, A and B are computed at the beginning time station of the time interval, in this case, time equals zero, using equations (22) and (25) respectively. One will note that all of the motion values needed to compute A and B are known; having been previously computed. The velocity, displacement, and acceleration of the parachute confluence point, at the end of the time interval, are then computed respectively using equations (26), (24), and (19) in that order. Using the acceleration value previously computed, updated estimates of the velocity, displacement, and acceleration are then obtained respectively from equations (21), (24), and (19). The

latest and previously computed values of displacement at the parachute confluence point are then compared. If the absolute value of the difference is greater than 0.00001 inches, velocity, displacement and acceleration values are updated, once again using equations (21), (24), and (19), and confluence point displacements are again compared. This procedure is repeated until the absolute value of the difference between these displacements is less than 0.00001 inches. At that point the nonlinear motion equation is considered solved for the assumed Δt.

Once the differential equation of motion has been solved for the assumed Δt value, the piston acceleration and piston displacement respectively are computed using equation (14a) and an equation analogous to equation (23). One will recall that the force exerted on the piston by the expanding gas, F, is not known as a function of time, but was computed from previously derived thermodynamic relationships at a specified value of piston displacement. Therefore, the specified and computed values of the piston displacement have to be compared, and the change in time, Δt, adjusted by means of a second iterative procedure in order to reconcile any differences. This process begins with an examination of the difference between displacement values, i.e., computed piston displacement minus the specified value. If this value is greater than the user-specified step increment, Δy, the Δt value assumed in the solution of the equation of motion is halved, and the equation is solved once again using the iterative procedure previously described. Specified and computed values of the piston displacement are compared and the process is repeated continuously until the difference is less than the step increment, Δy. At that point, updated values of Δt are obtained by means of extrapolation using the FORTRAN formula:

$$\Delta t = \Delta t - \Delta t \left(\frac{y_{DIFF}}{\Delta y} \right)$$

where y_{DIFF} is obtained by subtracting the specified value of the piston displacement from the computed value during each iteration[8]. The equal sign in the above FORTRAN formula by definition means "is replaced by". Thus, estimates of Δt are either increased or decreased prior to again solving the motion equation, depending respectively upon whether one has undershot (y_{DIFF} is negative) or overshot (y_{DIFF} is positive) the value of Δt during the previous iteration.

This two-step iterative procedure continues until the absolute value of y_{DIFF} is less than 0.00001 inches. At that point, the equation of motion is assumed to be

solved for that time step, having closely coupled the kinetic, kinematic and thermodynamic relationships on which system behavior is based. The remaining force, displacement, velocity, and acceleration values can now be computed. Payload acceleration is computed first using equation (14b). Piston and payload velocity are then computed using recurrence formulas analogous to equation (20); followed by the payload displacement using a recurrence formula similar to equation (23). The line tension, parachute force, retraction, retraction rate, and retraction acceleration are then obtained respectively, once again, from equations (12), (11), (15a), (15b), and (15c). Next, the elapsed time up to the time station is computed by adding Δt to the elapsed time at the previous time station. Finally, values are written to an output file, the piston displacement is increased by the user-specified step increment, Δy, and the two-step iterative procedure, described here, is carried out at the next time station, using the displacements, velocities, and accelerations from the most recently computed time station as initial conditions. The time history analysis continues one step at a time, at Δy increments, until either the time reaches the maximum input value specified or the piston displacement reaches its full stroke. The analysis is terminated once one of these parameters has been exceeded.

Comparison of Experimental Data with Model Predictions

To test the computer model's ability to predict system performance, FORTRAN model predictions were compared with experimental data obtained from tests conducted of a prototype PRSLS at Tustin MCAS in April 1994. A complete verification and validation of the computer model at the present time, however, is not possible. This is due to the fact that the Tustin experiments remain the only instrumented tests conducted to date on a PRSLS of this kind, and the amount of data gathered was limited. Therefore, only a cursory check of the computer model, based on this limited test data, can be made here.

Data recorded in the 20 April 1994 Tustin experiment are presented in Figure 3. This figure shows both the line tension and payload descent velocity plotted verses time. The retraction begins at approximately 7.5 seconds into the system's descent and ends at approximately 9.0 seconds; therefore, the total retraction takes approximately 1.5 seconds. One will observe that at 7.5 seconds, the line tension, or tension at the parachute confluence point, rises almost vertically from 62.5 pounds, the suspended weight, to about 100 pounds, and then decreases in a non-linear manner to about 50 pounds at 9.0 seconds. The payload velocity, meanwhile, decreases nonlinearly from the terminal descent velocity of approximately 21 feet per second at the beginning of retraction to around one-third that value at the end.

In order to compare FORTRAN computer model predictions with the experimental data from Tustin, the program was run with input parameters that closely approximate those of the prototype PRSLS tested at Tustin[9]. Input parameters specified in these runs were as follows:

Initial Pressure - 220 psig.
Tank Volume - 212 cubic inches
Piston Radius - 1.25 inches
Specific Heat Ratio - 1.4
Weight of Piston, Cross Member, and Top Pulleys - 1.0 pound
Weight of Payload and Retractor - 62.5 pounds
Piston Stroke - 24.0 inches
Step Increment - 0.1 inches
Gas Expansion Process: (1) Adiabatic (2) Isothermal
Parachute Diameter - 14 feet

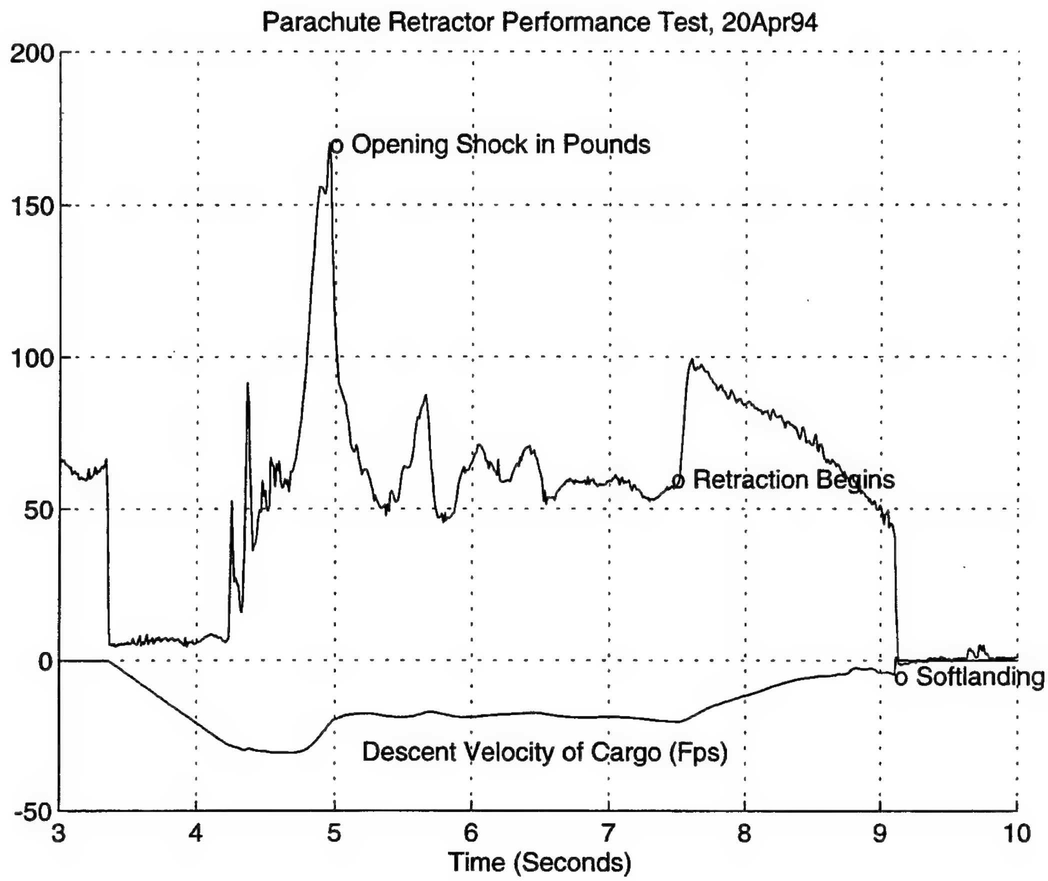

Figure 3: Tustin Experimental Data

Mechanical Advantage of Pulley System - 10
Drag Coefficient of Parachute - 0.75
Maximum Run Time - 4 seconds

The model was run twice; first assuming adiabatic expansion of the ideal gas, and then isothermal gas expansion. Although adiabatic expansion of the gas was expected to be closer to actual gas behavior, isothermal gas expansion was also investigated to determine whether appreciable difference between the two processes can be expected over the anticipated short duration in which retraction occurs. Model results were summarized in a series of plots using the MATLAB computer program[10]. A listing of this MATLAB program is shown in Appendix B. This program allows the user to selectively view results by choosing from a list of 20 available plots of various model output data. Model predictions of payload velocity and line tension vs. time are presented in the plots shown in Figure 4. Although no attempts were made to refine any of the model inputs, there appears to be close correlation between computer predictions assuming adiabatic gas expansion and the observed test behavior.

Although not shown in the line tension plot, one will observe that since the system is initially at terminal descent velocity, the tension at the parachute confluence point prior to retraction is equal to the suspended weight; in this case, 62.5 pounds. Therefore, at time equals zero, when retraction begins, the tension at the parachute confluence point is predicted to rise vertically from the suspended weight up to approximately 108 pounds. This overprediction of the peak tension at the start of retraction is undoubtedly due to the instantaneous start assumed in the development of the model. The predicted tension is then observed to decrease nonlinearly to 54.6 pounds at the end of the retraction. The payload velocity, meanwhile, is shown to decrease nonlinearly from the computed terminal descent velocity of 256 inches per second (21.3 feet per second) to a minimum of approximately 81.4 inches per second (6.8 feet per second). The payload velocity then increases slightly before the piston reaches its full stroke. Full retraction of the parachute is predicted to take place in just under 1.5 seconds. Thus, close agreement between experimental test data and model predictions assuming adiabatic gas expansion was obtained in terms of not only the magnitude of values, but also the shapes of the curves. One will further observe that significant differences between adiabatic and isothermal computer model predictions are obtained despite the short duration of the retraction. These differences become more pronounced toward the end of the retraction; a product of the isothermal process's larger amount

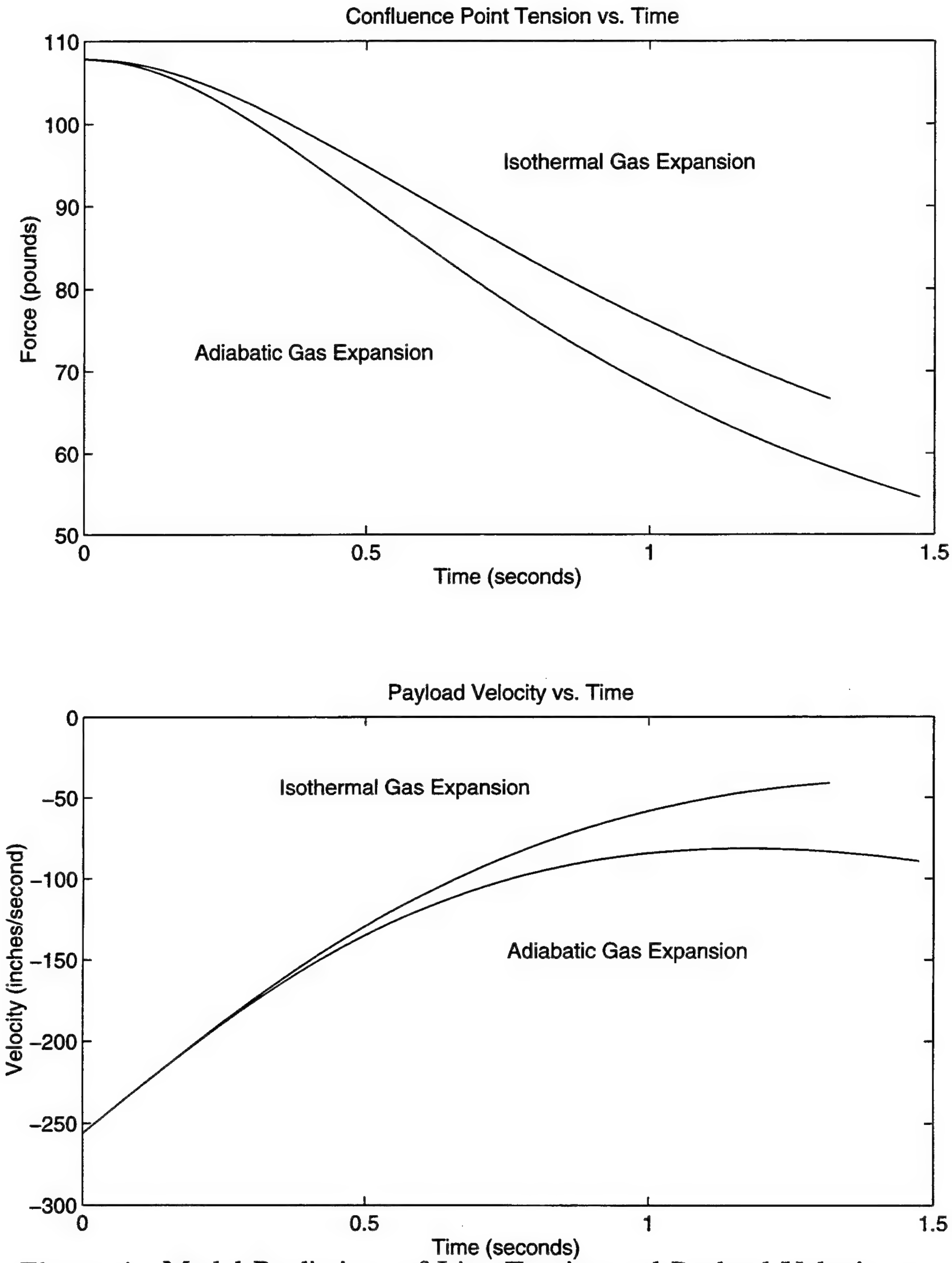

Figure 4: Model Predictions of Line Tension and Payload Velocity vs. Time

of available energy as compared with that of the adiabatic process.

The reader's attention is directed next to Figure 5, showing FORTRAN model predictions of both parachute force vs. time, and payload velocity vs. payload displacement. Though not directly used in the comparisons of experimental data with model predictions, Figure 5 is presented here, nonetheless, by way of example, to provide the reader a greater appreciation of the model's capabilities. The intent is to demonstrate the model's usefulness and flexibility, by showing how the computer model can be used by system developers and end users alike, and not just as a research tool. Plots of the parachute force, as well as other force and acceleration time histories provided by the model, for example, can be used by system developers to check and refine the design of the retractor mechanism and parachutes, prior to system fabrication and testing. Therefore, much of the costly, time consuming need to build, instrument and test prototypes, could be eliminated during the Engineering and Manufacturing Development (EMD) phase of system development, by using the model as a design tool. Similarly, given a PRSLS of some known capacity, one can quickly compute the pressure and activation height required to soft land a payload of some lesser weight from plots of the payload velocity vs. payload displacement. Therefore, the model potentially could be used as a simulation tool to support PRSLS airdrop operations by providing the end user, i.e., troops in the field, the ability to quickly determine soft landing requirements for cargo of varied weights.

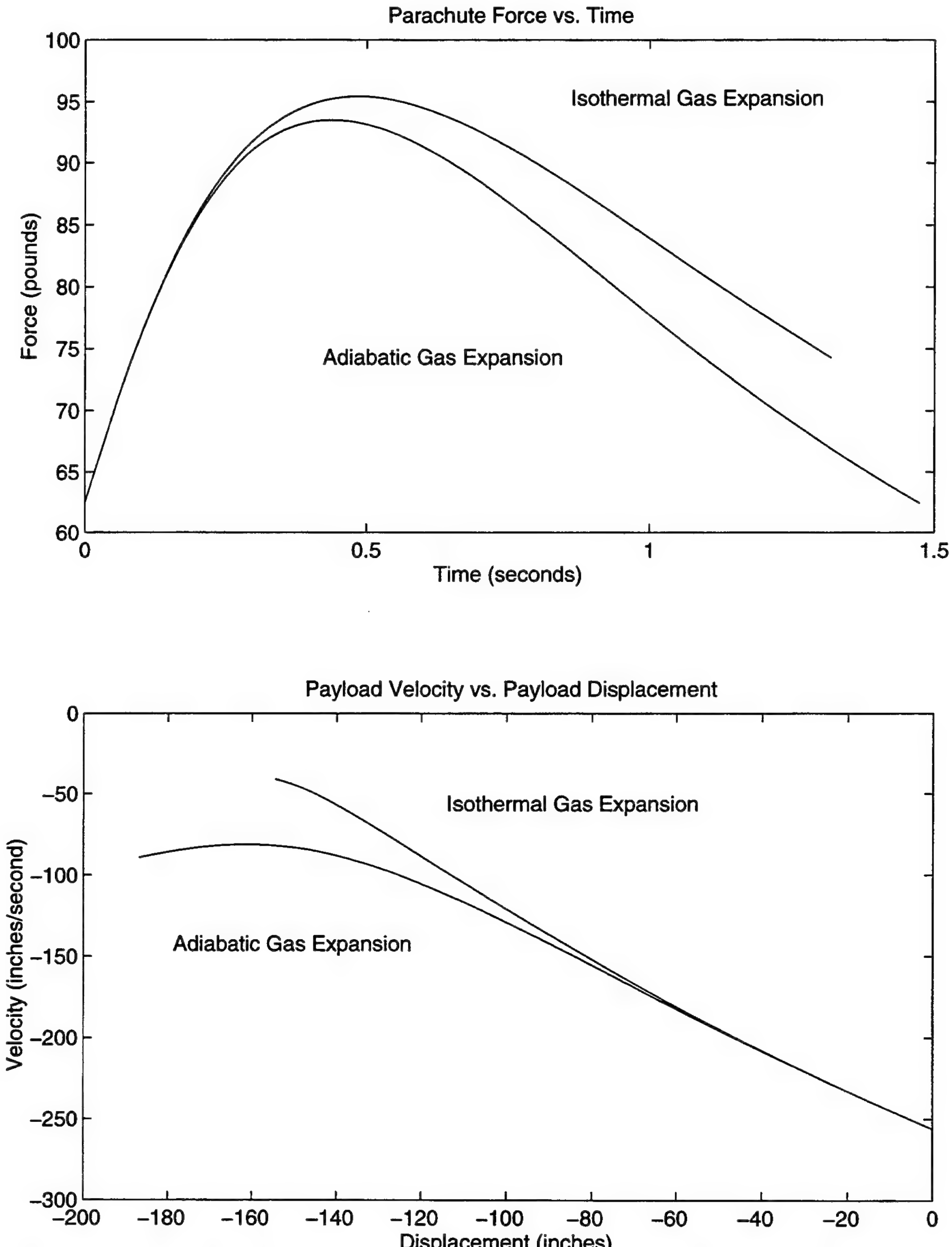

Figure 5: Model Predictions of Parachute Force vs. Time and Payload Velocity vs. Payload Displacement

Conclusions and Recommendations

This report has traced the development of a predictive, time history model of the NRDEC Parachute Retraction Soft Landing System (PRSLS); describing the model's underlying theory and current status. Based on comparisons of experimental data with model predictions, it was concluded that this computer model, in its current form, can predict, with a fair degree of accuracy, the system's motion and forces during retraction. The model, once validated, possesses the flexibility and capability to serve as a research, design, and simulation tool that will further our understanding of system behavior, simplify the design process, and aid in the selection of pressures and activation heights. As such, this model represents an enabling technology that can be used to speed introduction of the PRSLS into the Army's inventory. Therefore, it is recommended that more experimental data be obtained to validate the model over a range of payload weights and pulley ratios and/or provide a foundation for further model refinements.

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10. MATLAB High-Performance Numeric Computation and Visualization Software, The Math Works Inc. MATLAB is a technical computing environment for high-performance numeric computation and visualization.

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Appendices

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Appendix A

FORTRAN Listing of the Parachute Retraction Soft Landing System (PRSLS) Model

Appendix A

```
C                        PROGRAM: 'RETRACTOR'
C
C *****************************************************************
C THIS PROGRAM MODELS BOTH THE LANDING RETRACTOR MECHANISM AND THE
C PARACHUTE CANOPY.  OUTPUT INCLUDES A COMPLETE DESCRIPTION OF THE
C DYNAMIC MOTION OF THE SYSTEM INCLUDING DISPLACEMENT, VELOCITY AND
C ACCELERATION OF THE PISTON, PAYLOAD AND CANOPY, LINE TENSION,
C PARACHUTE FORCE, PISTON WORK, RETRACTION RATE, ETC. VS. TIME.
C *****************************************************************
      IMPLICIT DOUBLE PRECISION(A-H,J-M,O-Z),INTEGER (I,N)
      DIMENSION Y(0:5000),YC(0:5000),YP(0:5000),VY(0:5000),VYC(0:5000)
      DIMENSION VYP(0:5000),AY(0:5000),AYC(0:5000),AYP(0:5000)
      DIMENSION F(0:5000)
      DIMENSION PWORK(0:5000),T(0:5000),FPAR(0:5000),DL(0:5000)
      DIMENSION VL(0:5000),AL(0:5000),A(0:5000),B(0:5000)
C *****************************************************************
C TO EXECUTE PROGRAM TYPE 'RETRACTOR' AND HIT THE RETURN KEY
C TYPE IN VALUES FOR THE FOLLOWING VARIABLES SEPARATED EITHER BY
C A SPACE OR COMMA:
C
C           INITIAL PRESSURE IN PSI. (GAUGE)
C           INITIAL VOLUME IN CUBIC INCHES
C           PISTON RADIUS IN INCHES
C           SPECIFIC HEAT RATIO, k=Cp/Cv
C           WEIGHT OF PISTON, CROSS MEMBER AND TOP PULLEYS IN LBS.
C           WEIGHT OF THE PAYLOAD AND RETRACTOR IN LBS.
C           PISTON STROKE IN INCHES
C           INCREMENT OF Y IN INCHES
C           GAS EXPANSION PROCESS: (1) ADIABATIC (2) ISOTHERMAL
C           PARACHUTE DIAMETER IN FEET
C           MECHANICAL ADVANTAGE OF PULLEY SYSTEM (EVEN INTEGER)
C           DRAG COEFFICIENT OF PARACHUTE
C           MAXIMUM RUN TIME IN SECONDS
C *****************************************************************
      READ *,PIN,VIN,R,k,WPIST,WP,STROKE,DELTAY,IPROC,DPAR,N,CD,TMAX
C
C WRITES VARIABLE VALUES TO OUTPUT FILE
C
      OPEN(24,FILE='OUTPUT')
      WRITE(24,50)PIN
 50   FORMAT('INITIAL PRESSURE IN PSI.'/
     2       5X,F13.5)
      WRITE(24,51)VIN
 51   FORMAT('INITIAL VOLUME IN CUBIC INCHES'/
     2       5X,F13.5)
      WRITE(24,52)R
 52   FORMAT('PISTON RADIUS IN INCHES'/
     2       5X,F13.5)
      WRITE(24,53)k
 53   FORMAT('SPECIFIC HEAT RATIO, k=Cp/Cv'/
     2       5X,F13.5)
      WRITE(24,54)WPIST
 54   FORMAT('WEIGHT OF PISTON, CROSS MEMBER AND TOP PULLEYS IN LBS.'/
     2       5X,F13.5)
```

```
      WRITE(24,55)WP
 55   FORMAT('WEIGHT OF THE PAYLOAD AND RETRACTOR IN LBS.'/
     2       5X,F13.5)
      WRITE(24,56)STROKE
 56   FORMAT('PISTON STROKE IN INCHES'/
     2       5X,F13.5)
      WRITE(24,57)DELTAY
 57   FORMAT('INCREMENT OF Y IN INCHES'/
     2       5X,F13.5)
      IF(IPROC.EQ.1) THEN
      WRITE(24,58)
 58   FORMAT('ADIABATIC GAS EXPANSION PROCESS')
      GO TO 61
      ELSE
      CONTINUE
      ENDIF
      IF(IPROC.EQ.2) THEN
      WRITE(24,60)
 60   FORMAT('ISOTHERMAL GAS EXPANSION PROCESS')
      GO TO 61
      ELSE
      GO TO 31
      ENDIF
 61   WRITE(24,62)DPAR
 62   FORMAT('PARACHUTE DIAMETER IN FEET'/
     2       5X,F13.5)
      WRITE(24,63)N
 63   FORMAT('MECHANICAL ADVANTAGE OF PULLEY SYSTEM'/
     2       5X,I4)
      WRITE(24,64)CD
 64   FORMAT('DRAG COEFFICIENT OF PARACHUTE'/
     2       5X,F13.5)
      WRITE(24,65)TMAX
 65   FORMAT('MAXIMUM RUN TIME IN SECONDS'/
     2       5X,F13.5)
C
C DEFINE AND COMPUTE CONSTANT TERMS USED IN THE DIFFERENTIAL
C EQUATIONS OF MOTION
C
      g=32.174*12.0
      pi=3.141592654
      PA=14.7
      RPAR=DPAR/2.
      WAIR=0.07648
      rho=WAIR/32.174
      VTERM=12.0*SQRT(2.0*WP/(pi*RPAR**2.*CD*rho))
      WP=WP-WPIST
      MP=WP/g
      MPIST=WPIST/g
      MC=(2.*pi*rho*RPAR**3.)/(3.*12.)
      C1=pi*RPAR**2.*CD*rho/(2.*144.)
      C2=N**2*C1*MP+(N+1)*C1*MPIST
      C3=(N+1)*MC*MPIST+MP*MPIST+N**2*MC*MP
      C4=(N-1)*g*MPIST*MP
      C5=N*MP+MPIST
      PIN=PIN+PA
```

```
C
C BEGINNING OF COMPUTATIONS AT FIRST TIME STATION, Y(0)=0., TIME=0.
C
      YPIST=0.
      I=0
      TIME=0.
      Y(I)=YPIST
      YC(I)=0.
      YP(I)=0.
      VY(I)=0.
      VYC(I)=VTERM
      VYP(I)=-VTERM
      PWORK(I)=0.
      IF(IPROC.EQ.1) THEN
      F(I)=(PIN*VIN**k*pi*R**2./(VIN+pi*R**2.*Y(I))**k)-PA*pi*R**2.
      GO TO 11
      ELSE
      GO TO 10
      ENDIF
 10   F(I)=(PIN*VIN*pi*R**2./(VIN+pi*R**2.*Y(I)))-PA*pi*R**2.
 11   AYC(I)=(C5*F(I)-C2*VYC(I)**2.-C4)/C3
      AY(I)=(F(I)-N*C1*VYC(I)**2.-N*MC*AYC(I)-WPIST)/MPIST
      AYP(I)=((N+1)*C1*VYC(I)**2.+(N+1)*MC*AYC(I)-F(I)-WP)/MP
      T(I)=C1*VYC(I)**2.+MC*AYC(I)
      FPAR(I)=C1*VYC(I)**2.
      DL(I)=YC(I)+YP(I)
      VL(I)=VYC(I)+VYP(I)
      AL(I)=AYC(I)+AYP(I)
      GO TO 30
C
C  END OF COMPUTATIONS AT FIRST TIME STATION, Y(0)=0., TIME=0.
C
 20   IF(IPROC.EQ.1) THEN
      F(I)=(PIN*VIN**k*pi*R**2./(VIN+pi*R**2.*Y(I))**k)-PA*pi*R**2.
      PWORK(I)=(PIN*VIN**k*(VIN+pi*R**2.*Y(I))**(1-k)-PIN*VIN)/(1-k)
      PWORK(I)=PWORK(I)-PA*pi*R**2.*Y(I)
      GO TO 22
      ELSE
      GO TO 21
      ENDIF
 21   F(I)=(PIN*VIN*pi*R**2./(VIN+pi*R**2.*Y(I)))-PA*pi*R**2.
      PWORK(I)=PIN*VIN*DLOG((VIN+pi*R**2.*Y(I))/VIN)
      PWORK(I)=PWORK(I)-PA*pi*R**2.*Y(I)
 22   DELTAT=0.1
 25   A(I-1)=VYC(I-1)+AYC(I-1)*DELTAT/2.
      B(I-1)=YC(I-1)+2.*VYC(I-1)*DELTAT/3.+AYC(I-1)*DELTAT**2./6.
      VYC(I)=VYC(I-1)+AYC(I-1)*DELTAT
      YC(I)=B(I-1)+VYC(I)*DELTAT/3.
      AYC(I)=(C5*F(I)-C2*VYC(I)**2.-C4)/C3
      PREVYC=YC(I)
 23   VYC(I)=A(I-1)+AYC(I)*DELTAT/2.
      YC(I)=B(I-1)+VYC(I)*DELTAT/3.
      AYC(I)=(C5*F(I)-C2*VYC(I)**2.-C4)/C3
      IF((DABS(YC(I)-PREVYC)).LT.0.00001) THEN
      GO TO 24
      ELSE
```

```
      PREVYC=YC(I)
      GO TO 23
      ENDIF
 24   AY(I)=(F(I)-N*C1*VYC(I)**2.-N*MC*AYC(I)-WPIST)/MPIST
      Y(I)=Y(I-1)+VY(I-1)*DELTAT+(2.*AY(I-1)+AY(I))*DELTAT**2./6.
      YDIFF=Y(I)-YPIST
      IF(YDIFF .GE. DELTAY) THEN
      DELTAT=DELTAT/2.
      GO TO 25
      ELSE
      CONTINUE
      ENDIF
      IF((YDIFF.LT.DELTAY).AND.((DABS(YDIFF)).GT.0.00001)) THEN
      DELTAT=DELTAT-DELTAT*(YDIFF/DELTAY)
      GO TO 25
      ELSE
      VY(I)=VY(I-1)+DELTAT/2.*(AY(I)+AY(I-1))
      AYP(I)=((N+1)*C1*VYC(I)**2.+(N+1)*MC*AYC(I)-F(I)-WP)/MP
      VYP(I)=VYP(I-1)+(AYP(I-1)+AYP(I))*DELTAT/2.
      YP(I)=YP(I-1)+VYP(I-1)*DELTAT+(2.*AYP(I-1)+AYP(I))*DELTAT**2./6.
      T(I)=C1*VYC(I)**2.+MC*AYC(I)
      FPAR(I)=C1*VYC(I)**2.
      DL(I)=YC(I)+YP(I)
      VL(I)=VYC(I)+VYP(I)
      AL(I)=AYC(I)+AYP(I)
      TIME=TIME+DELTAT
      ENDIF
C
C WRITE RESULTS TO OUTPUT FILE AND COMPARE TIME AND PISTON
C DISPLACEMENT RESPECTIVELY TO MAXIMUM TIME AND PISTON STROKE
C
 30   IF(TIME .GT. TMAX) GO TO 31
      WRITE(24,66)I,F(I),DELTAT,TIME,Y(I),YC(I),YP(I),AY(I),AYC(I),
     2AYP(I),VY(I),VYC(I),VYP(I),PWORK(I),T(I),FPAR(I),
     3DL(I),VL(I),AL(I)
 66   FORMAT(2X,I5,18(2X,E13.5))
      IF(Y(I) .LT. STROKE) THEN
      I=I+1
      YPIST=YPIST+DELTAY
      Y(I)=YPIST
      GO TO 20
      ELSE
      CONTINUE
      ENDIF
 31   CLOSE(24,STATUS='KEEP')
      STOP
      END
```

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Appendix B

MATLAB Program Listing

Appendix B

MATLAB Program Listing

```
clear
load -ascii PLR1.mat
load -ascii PLR2.mat
who
clg
hold off
k = menu('CHOOSE PLOT',...
'PISTON ACCELERATION VS. TIME',...
'CONFLUENCE POINT ACCELERATION VS. TIME',...
'PAYLOAD ACCELERATION VS. TIME',...
'PISTON VELOCITY VS. TIME',...
'CONFLUENCE POINT VELOCITY VS. TIME',...
'PAYLOAD VELOCITY VS. TIME',...
'PISTON DISPLACEMENT VS. TIME',...
'CONFLUENCE POINT DISPLACEMENT VS. TIME',...
'PAYLOAD DISPLACEMENT VS. TIME',...
'PISTON WORK VS. RETRACTION',...
'PISTON FORCE VS. DISPLACEMENT',...
'PISTON WORK VS. DISPLACEMENT',...
'PISTON FORCE VS. TIME',...
'PISTON WORK VS. TIME',...
'CONFLUENCE POINT TENSION VS. TIME',...
'PARACHUTE FORCE VS. TIME',...
'RETRACTION VS. TIME',...
'RETRACTION VELOCITY VS. TIME',...
'RETRACTION ACCELERATION VS. TIME',...
'PAYLOAD VELOCITY VS. PAYLOAD DISPLACEMENT',...
'QUIT')
if k == 1
clg
plot(PLR1(:,4),PLR1(:,8),'-r',PLR2(:,4),PLR2(:,8),'-w')
title('PISTON ACCELERATION VS. TIME')
xlabel('TIME SECONDS')
ylabel('ACCELERATION INCHES PER SEC 2')
%
elseif k == 2
clg
plot(PLR1(:,4),PLR1(:,9),'-r',PLR2(:,4),PLR2(:,9),'-w')
title('CONFLUENCE POINT ACCELERATION VS. TIME')
xlabel('TIME SECONDS')
ylabel('ACCELERATION INCHES PER SEC 2')
%
elseif k == 3
clg
plot(PLR1(:,4),PLR1(:,10),'-r',PLR2(:,4),PLR2(:,10),'-w')
title('PAYLOAD ACCELERATION VS. TIME')
xlabel('TIME SECONDS')
ylabel('ACCELERATION INCHES PER SEC 2')
%
elseif k == 4
clg
plot(PLR1(:,4),PLR1(:,11),'-r',PLR2(:,4),PLR2(:,11),'-w')
```

```
title('PISTON VELOCITY VS. TIME')
xlabel('TIME SECONDS')
ylabel('VELOCITY INCHES PER SEC')
%
elseif k == 5
clg
plot(PLR1(:,4),PLR1(:,12),'-r',PLR2(:,4),PLR2(:,12),'-w')
title('CONFLUENCE POINT VELOCITY VS. TIME')
xlabel('TIME SECONDS')
ylabel('VELOCITY INCHES PER SEC')
%
elseif k == 6
clg
plot(PLR1(:,4),PLR1(:,13),'-r',PLR2(:,4),PLR2(:,13),'-w')
title('PAYLOAD VELOCITY VS. TIME')
xlabel('TIME SECONDS')
ylabel('VELOCITY INCHES PER SEC')
%
elseif k == 7
clg
plot(PLR1(:,4),PLR1(:,5),'-r',PLR2(:,4),PLR2(:,5),'-w')
title('PISTON DISPLACEMENT VS. TIME')
xlabel('TIME SECONDS')
ylabel('DISPLACEMENT INCHES')
%
elseif k == 8
clg
plot(PLR1(:,4),PLR1(:,6),'-r',PLR2(:,4),PLR2(:,6),'-w')
title('CONFLUENCE POINT DISPLACEMENT VS. TIME')
xlabel('TIME SECONDS')
ylabel('DISPLACEMENT INCHES')
%
elseif k == 9
clg
plot(PLR1(:,4),PLR1(:,7),'-r',PLR2(:,4),PLR2(:,7),'-w')
title('PAYLOAD DISPLACEMENT VS. TIME')
xlabel('TIME SECONDS')
ylabel('DISPLACEMENT INCHES')
%
elseif k == 10
clg
plot(PLR1(:,17),PLR1(:,14),'-r',PLR2(:,17),PLR2(:,14),'-w')
title('PISTON WORK VS. RETRACTION')
xlabel('RETRACTION INCHES')
ylabel('WORK INCH POUNDS')
%
elseif k == 11
clg
plot(PLR1(:,5),PLR1(:,2),'-r',PLR2(:,5),PLR2(:,2),'-w')
title('PISTON FORCE VS. DISPLACEMENT')
xlabel('DISPLACEMENT INCHES')
ylabel('FORCE POUNDS')
%
elseif k == 12
clg
plot(PLR1(:,5),PLR1(:,14),'-r',PLR2(:,5),PLR2(:,14),'-w')
```

```
title('PISTON WORK VS. DISPLACEMENT')
xlabel('DISPLACEMENT INCHES')
ylabel('WORK INCH|POUNDS')
%
elseif k == 13
clg
plot(PLR1(:,4),PLR1(:,2),'-r',PLR2(:,4),PLR2(:,2),'-w')
title('PISTON FORCE VS. TIME')
xlabel('TIME SECONDS')
ylabel('FORCE POUNDS')
%
elseif k == 14
clg
plot(PLR1(:,4),PLR1(:,14),'-r',PLR2(:,4),PLR2(:,14),'-w')
title('PISTON WORK VS. TIME')
xlabel('TIME SECONDS')
ylabel('WORK INCH|POUNDS')
%
elseif k == 15
clg
plot(PLR1(:,4),PLR1(:,15),'-r',PLR2(:,4),PLR2(:,15),'-w')
title('CONFLUENCE POINT TENSION VS. TIME')
xlabel('TIME SECONDS')
ylabel('TENSION POUNDS')
%
elseif k == 16
clg
plot(PLR1(:,4),PLR1(:,16),'-r',PLR2(:,4),PLR2(:,16),'-w')
title('PARACHUTE FORCE VS. TIME')
xlabel('TIME SECONDS')
ylabel('FORCE POUNDS')
%
elseif k == 17
clg
plot(PLR1(:,4),PLR1(:,17),'-r',PLR2(:,4),PLR2(:,17),'-w')
title('RETRACTION VS. TIME')
xlabel('TIME SECONDS')
ylabel('RETRACTION INCHES')
%
elseif k == 18
clg
plot(PLR1(:,4),PLR1(:,18),'-r',PLR2(:,4),PLR2(:,18),'-w')
title('RETRACTION VELOCITY VS. TIME')
xlabel('TIME SECOND')
ylabel('VELOCITY INCHES PER SEC')
%
elseif k == 19
clg
plot(PLR1(:,4),PLR1(:,19),'-r',PLR2(:,4),PLR2(:,19),'-w')
title('RETRACTION ACCELERATION VS. TIME')
xlabel('TIME SECONDS')
ylabel('ACCELERATION INCHES PER SEC 2')
%
elseif k == 20
clg
plot(PLR1(:,7),PLR1(:,13),'-r',PLR2(:,7),PLR2(:,13),'-w')
```

```
title('PAYLOAD VELOCITY VS. PAYLOAD DISPLACEMENT')
xlabel('DISPLACEMENT INCHES')
ylabel('VELOCITY INCHES PER SEC')
end
```

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List of Symbols

C_D	parachute drag coefficient
F	force exerted on piston by expanding gas
$F_{Parachute}$	parachute force
H	enthalpy
L	retraction, i.e., $L=Ny=y_C+y_P$
M_C	parachute apparent mass
M_P	mass of payload and retractor components that move with payload
M_{PIST}	mass of piston and retractor components that move with piston
N	mechanical advantage
P	pressure
P_i	initial tank pressure
P_1	initial pressure
P_2	final pressure
Q	heat
R	universal gas constant, where $R = c_p - c_v$
S	piston stroke
T	temperature
T_1	initial temperature
T_2	final temperature
T_C	tension at parachute confluence point
U	internal energy
V	volume
V_1	initial volume
V_2	final volume
V_i	tank volume
W	work
W_q	adiabatic work
W_T	isothermal work
W_C	weight of air in the parachute; assumed negligible
W_P	weight of payload and retractor components that move with payload
W_{PIST}	weight of piston and retractor components that move with piston
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
k	specific heat ratio, c_p/c_v
n	number of moles of gas
r	piston radius

r_P	nominal parachute radius
t	time
v_{TERM}	terminal descent velocity
x	some generalized displacement
y	piston displacement
y_C	displacement at parachute confluence point
y_P	payload displacement
y_{DIFF}	computed minus specified piston displacement
$\Delta H_{\text{Ideal Gas}}$	change in enthalpy of an ideal gas
$\Delta U_{\text{Ideal Gas}}$	change in internal energy of an ideal gas
Δt	change in time over interval
Δy	input step increment of piston displacement
ρ	density of air

Throughout this report, a dot or double dot notation over any of the displacements (x, y, y_C , y_P or L) will be used to designate the first and second derivatives of that displacement with respect to time, or in other words, the velocity and acceleration.